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B.Sc Part II PAPER-II

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Th B. The inverse of any element of a subgroup is the same as the inverse of the same regarded as an element of the group.

Let e be the identity of G as well as of H

Let $a \in H$. Suppose b is the inverse of a in H and c is the inverse of a in G. Then we have $ba = e$ and $ca = e$.
$\therefore$ in G we have $ba = ca \Rightarrow b = c$

Th C A non-empty subset H of a group G is a subgroup of G iff

(a) $a, b \in H \Rightarrow ab \in H$

(b) $a \in H \Rightarrow a^{-1} \in H$

Let H be a subgroup of G then by definition it follows that (a) & (b) hold. conversely, let the given conditions hold in H

Closure holds in H by G.

again $a, b, c \in H \Rightarrow a, b, c \in G$

$\Rightarrow a(bc) = (ab)c$

Hence associativity holds in H.

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Also for any $a \in H$, $a^{-1} \in H$ and so by (a)

$$aa^{-1} \in H \Rightarrow e \in H$$

Thus H has identity.
Inverse of each element of H is in H by (b)
Hence H satisfies all conditions in the definition of a group and thus it forms a group and therefore a subgroup of G.